

AN INFORMATION-THEORETIC ANALYSIS OF CLINICAL REFRACTION*

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ABSTRACT

Clinical refraction may be viewed as a procedure for generating messages—prescriptions—each containing a certain definite amount of information acquired from the patient. As part of the development of an automatic refraction system, this information flow has been analyzed, using concepts and metrics introduced by Shannon. Results from a sample block of 650 consecutive patients entering a University optometric clinic indicate that the general use prescription for both eyes contains about 23 bits of information. Total information acquired on sex, age, visual acuity, static skiametry and subjective refraction was about 60 bits, indicating 62% redundancy. Detailed analysis showed that subjective refraction could alone supply 75% of the required information, and that the inclusion of either age or visual acuity data increased this to 85%. The missing 15% of the prescription may either be genuine information derived from unrecorded or multiple sources ("clinical judgment") or spurious information (error or "noise"). Hence minimum criteria are established for the informational performance of an automatic refraction system.

Prerequisite to the design of an efficient process or system of any kind is a performance specification stating what it must accomplish, under what constraints, and in what environment. We consider here the clinical refraction

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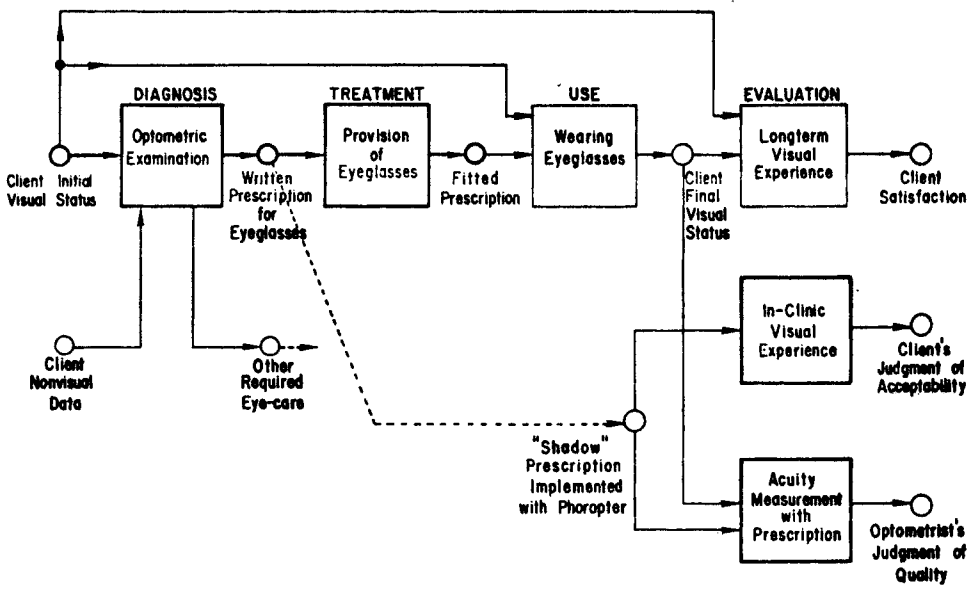


Fig. 1. Partial Operation Flow-Chart for Clinical Optometry.

process. As reported in earlier papers^{1, 2}, we are currently attempting to develop a computer-based automatic system for performing refractions, and in this connection we experienced a need for a more precise performance specification than appeared to be available in the literature. The present paper reports a quantitative study, based on information theory, aimed at filling this need and thus providing a rational basis for the design of the automatic system.

Fig. 1 gives an operation process chart for the over-all eye care system. Two of the four processes carried out within the optometric clinic itself, viz., "diagnosis" and "treatment," are linked solely by a message, viz., the written prescription. Thus diagnosis may be characterized as an information process—acquiring and processing data from the client—while treatment is a physical production process. Since eyeglasses can be produced with great precision, the quality of the written message emerging from diagnosis is apparently assessed at a later stage by the client's ultimate ability to evaluate his visual experience and express satisfaction (or dissatisfaction) with the result. It follows that the correctness of each prescription should ideally be established by observing long period patient satisfaction. Since this is not normally feasible, visual acuity with prescription appears to be used as the operational quality standard. Since this is established within the examination itself, and lacking direct evidence of client satisfaction, we assumed that recorded prescriptions exceeded the required minimum quality standard. While they may sometimes have been more precise than strictly needed, it seemed reasonable to take present practice as a criterion. Interpreted in terms of a performance specification this meant that our proposed automatic system must produce prescriptions agreeing with those produced by the present (manual) system. Study of current practice was therefore undertaken to provide a design target for the automatic system.

Observation and time measurement of typical eye examinations provided

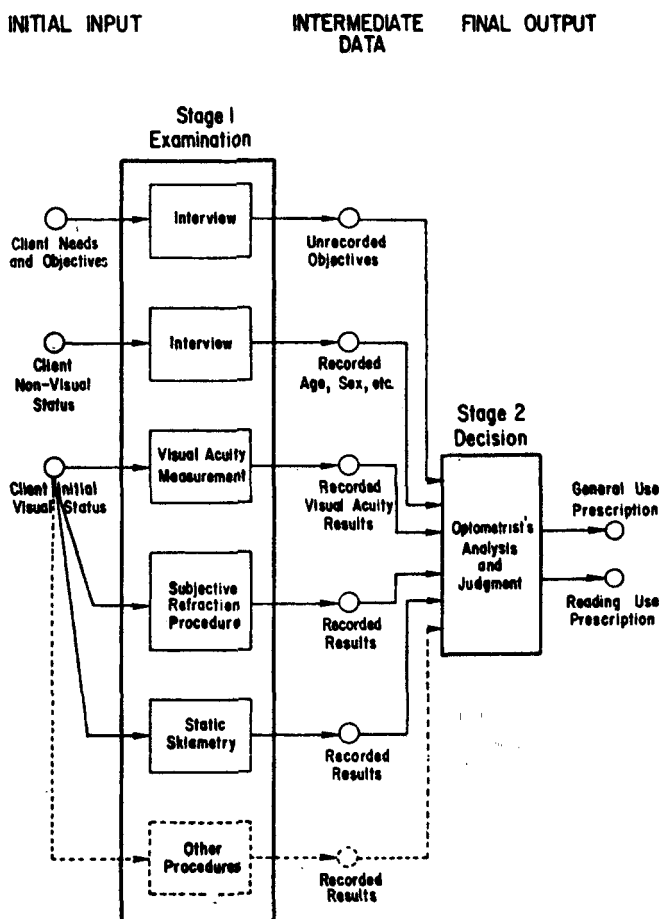


Fig. 2. Information-Transfer Model for the Diagnostic Stages of Clinical Optometry.

a partial specification of the manual process, but failed to define its quantitative objectives. The question also arose, which parts of the observed procedure could be deemed essential.

We, therefore, undertook to measure the diagnostic information process as it occurs in a specific clinical situation, viz., the University of California Optometric Clinic, with the following objectives: 1) to determine how much information is contained in a single prescription; 2) to measure how much information each type of intermediate data available to the optometrist contributes to the final prescription; and, 3) if possible, to identify a subset of non-redundant intermediate data sufficient to produce a minimally accurate prescription.

While various statistical methods such as multiple correlation could have been used to analyze the results, the theory of selective information originally developed by Shannon³ offered a more conceptually meaningful approach⁷ with results more directly applicable to computer-system design⁸. While op-

⁸The reader unfamiliar with information theory will find treatments at progressive levels of difficulty in References 11, 6, 3.

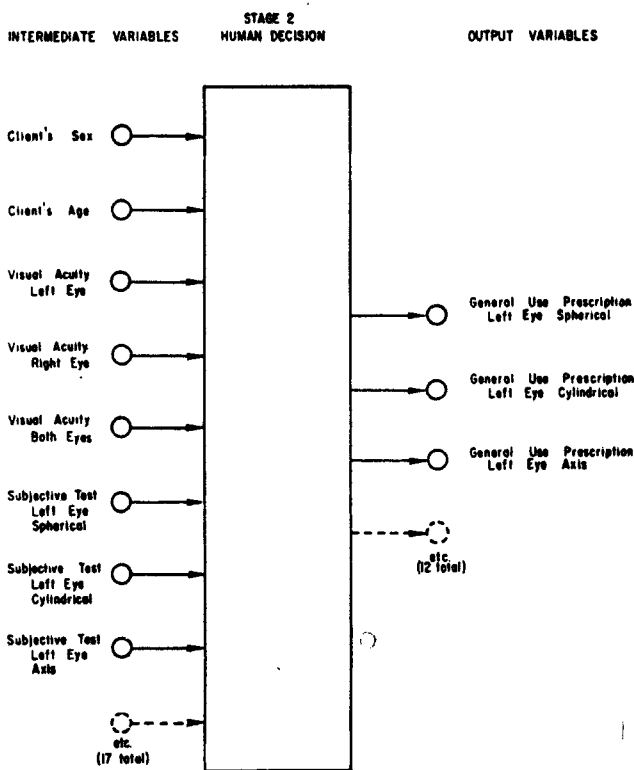


Fig. 3a. Possible Information-Transfer Models for the Decision Stage of Optometric Diagnosis. Multi-Input, Multi-Output Model.

ometric data are expressed in real numbers, the values of variables are generally restricted to a limited number of preferred values, and the discrete rather than continuous form of the theory was therefore employed⁴.

AN INFORMATION-THEORETIC MODEL OF EYE EXAMINATION

Fig. 2 shows an expanded version of the diagnostic process within the over-all eye care process. Stage 1 consists of an interview and three major procedures (visual acuity determination, static skiametry, subjective refraction) which are independent in the sense that none of them are prerequisite to others; results of all three are recorded along with various non-visual information. In Stage 2 the final prescription is developed from a knowledge of the various results of Stage 1. This process is performed by the optometrist without external aids, and is essentially unobservable.

Each of the unit processes identified in Fig. 2 can be regarded as a channel transmitting information from left to right, the over-all objective being to generate an output message (the prescription) accurately reflecting client initial visual status and objectives.

For any such channel, with input X and output Y, we may define (3) an input entropy H(X), output entropy H(Y), joint entropy H(X, Y) and transmission rate per message

$$(1) \quad R = H(X) + H(Y) - H(X, Y)$$

where entropy is defined as the mean logarithm of the probability of each possible message, for example

$$(2) \quad H(X) = - \sum_{i=1}^n p_i \log_2 p_i \text{ (bits)}$$

The subscripts i identify each of the n mutually exclusive possible messages in the set X and

$$(3) \quad \sum_{i=1}^n p_i = 1.$$

We have no access to the information sources (initial input) on the left-hand side of Fig. 2, but we may reasonably assume that the optometrist performs the prescribed procedures accurately, hence that there is perfect transmission of information from client initial visual status, etc., to the various intermediate recorded results of examination. Thus the intermediate data column should accurately reflect client visual status.

Not all the intermediate data acquired in the present procedure appear to contribute materially to the prescription, and the question of major interest in the present context concerned Stage 2, the decision stage. We were specifically interested in the relative weights assigned to the various intermediate data in generating the final prescriptions. Ideally the optometrist would be modelled as a 17 input/12 output channel in this case, since each prescription for each eye contains three data (sphere, cylinder, axis) and each of several intermediate results has a like dimensionality (Table 1). However, under known human information processing limitations^{4, 5, 8} it is most unlikely that all of the possible interactions are significant. We may therefore hope to approximate Stage 2 by a model consisting of a set of non-interacting parallel channels of which few transfer any information, as shown in Fig. 3b. Resource constraints did not permit full investigation of the total decision process, and attention was confined to the 17 intermediate results that seemed *prima facie* most likely to yield high information transfer to the final prescription. Even this leaves great com-

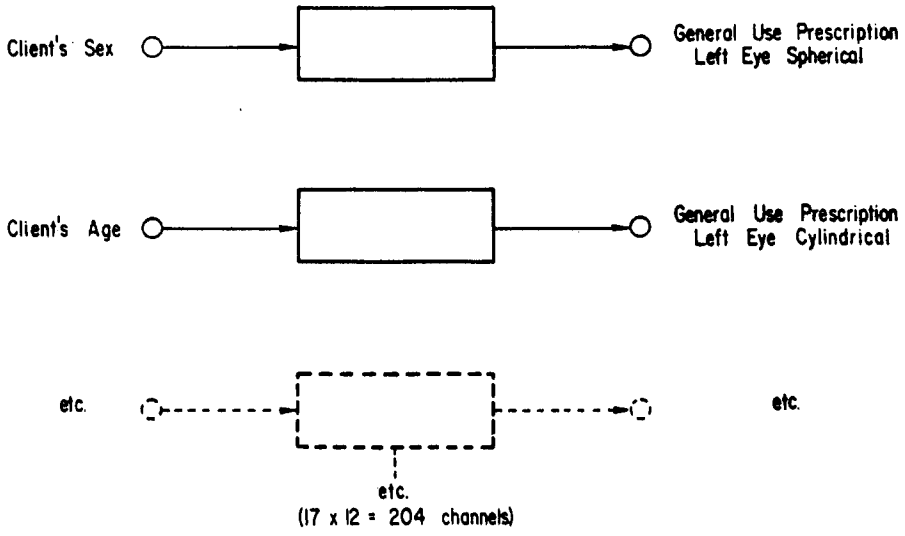


Fig. 3b. Simplified Parallel-Channel Model Including All Input/Output Pairs.

plexity, and further simplification by elimination of all channels except those linking equivalent data in intermediate results and prescription would be ideal. The study to be described shows that this is a reasonable approximation to the actual human decision process, but that there is some cross coupling between channels. The resulting model is shown in Fig. 3c.

TABLE 1
SUMMARY OF RECORDED DATA FROM WHICH INFORMATION-TRANSFER ESTIMATES WERE DERIVED

Number	Variable	Mean	Standard Deviation	Units	Maximum Entropy (bits)	Estimated Actual Entropy (bits)	Redundancy %
1	Sex	27.85	12.91	Years	1.00	0.99	1.0
2	Age				3.46	2.69	22.3
Intermediate Results							
3	Visual Acuity (Left Eye)	123.74	161.71	20/ft.	3.70	3.47	6.2
4	Visual Acuity (Right Eye)	121.58	163.52	20/ft.	3.70	3.53	17.0
5	Visual Acuity (Both Eyes)	105.56	153.52	20/ft.	3.70	3.14	15.1
6	Static Skiametry Spherical Error	-0.786	2.11	Diopters	5.13	4.79	6.6
7	Static Skiametry Cylindrical Error	-0.816	1.35	Diopters	3.46	2.65	23.4
8	Static Skiametry Axis Position	114.31	60.30	Degrees	5.17	3.92	24.2
9	Subjective Test Spherical Error	-0.863	2.13	Diopters	5.13	4.83	5.8
10	Subjective Test Cylindrical Error	-0.821	1.61	Diopters	3.46	2.62	24.3
11	Subjective Test Axis Position	110.28	59.74	Degrees	5.17	3.82	26.1
12	Subjective Test Spherical Error	-0.851	2.33	Diopters	5.13	4.80	6.4
13	Subjective Test Cylindrical Error	0.343	14.19	Diopters	3.46	2.63	24.0
14	Subjective Test Axis Position	113.55	174.31	Degrees	5.17	4.77	7.7
15	General Use Prescription Spherical Error	-0.943	2.23	Diopters	5.13	4.81	6.2
16	General Use Prescription Cylindrical Error	-0.137	10.06	Diopters	3.46	2.63	24.0
17	General Use Prescription Axis Position	99.02	110.11	Degrees	5.17	4.38	15.3
Output Data							
18	General Use Prescription Spherical Error	-1.164	2.22	Diopters	5.13	4.82	6.0
19	General Use Prescription Cylindrical Error	-0.579	0.74	Diopters	3.46	2.30	33.5
20	General Use Prescription Axis Position	73.51	65.38	Degrees	5.17	4.26	17.6
21	Reading Use Prescription Spherical Error	-0.863	2.44	Diopters	5.13	4.85	5.5
22	Reading Use Prescription Cylindrical Error	-0.821	0.66	Diopters	3.46	2.28	34.1
23	Reading Use Prescription Axis Position	110.28	63.85	Degrees	5.17	4.34	16.1
24	Reading Use Prescription Spherical Error	1.862	2.14	Diopters	5.13	3.82	25.5
25	Reading Use Prescription Cylindrical Error	-0.521	0.60	Diopters	3.46	2.19	36.7
26	Reading Use Prescription Axis Position	76.71	61.64	Degrees	5.17	4.38	15.3
27	Reading Use Prescription Spherical Error	1.905	2.30	Diopters	5.13	4.12	19.7
28	Reading Use Prescription Cylindrical Error	-0.560	0.64	Diopters	3.46	2.29	33.8
29	Reading Use Prescription Axis Position	72.25	60.10	Degrees	5.17	4.33	16.2

PROCEDURE

Records of refractions performed at the Optometry Clinic, School of Optometry, University of California, Berkeley, were made available to us^b. These were handwritten on a standard card. A random block of cards representing 650 sequential patients was selected for examination, representing a 12-month period in 1967. The 29 selected variables were transcribed from the record cards onto IBM cards along with a serial number used for identification.

The following data were transcribed:

1) *Patient non-visual status*—sex, age (data #1-2)

2) *Intermediate results of eye examination*

a) Visual acuity (data #3-5)—determined by Snellen Chart for each eye separately and for both together, describing the initial refractive state of the patient.

b) Objective refraction (2 x 3 dimensions; data #6-11). Refractive error obtained by static skiametry—these are measurements taken by the refractionist without requiring the client to use his vision actively.

c) Subjective refraction (2 x 3 dimensions; data #12-17). Refractive error obtained by subjective refraction using the phoropter—these require active cooperation of the client using visual discrimination and verbal report. The refractionist makes phoropter settings and interprets the verbal response.

3) *The prescription* (data #18-29)—The prescription for each client includes three dimensions of correction for general use for each eye (axis direction, cylindrical correction power, and spherical correction power); in about 100 cases, associated with age, a reading use prescription was recorded with the same dimensions.

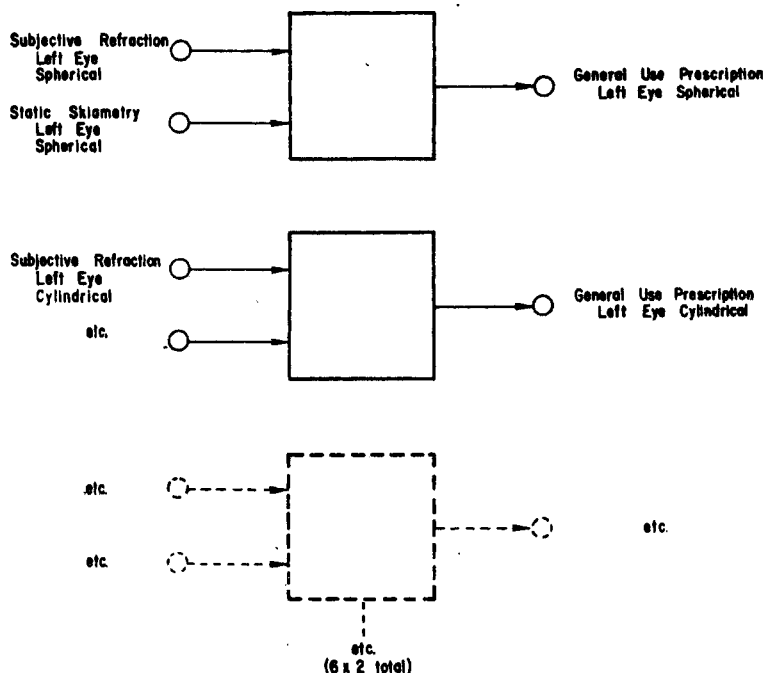


Fig. 3c. Further Simplified Model Including Only Six Parallel Channels, Each 2 Input 1 Output.

^bBy kind permission of the Dean of the School of Optometry.

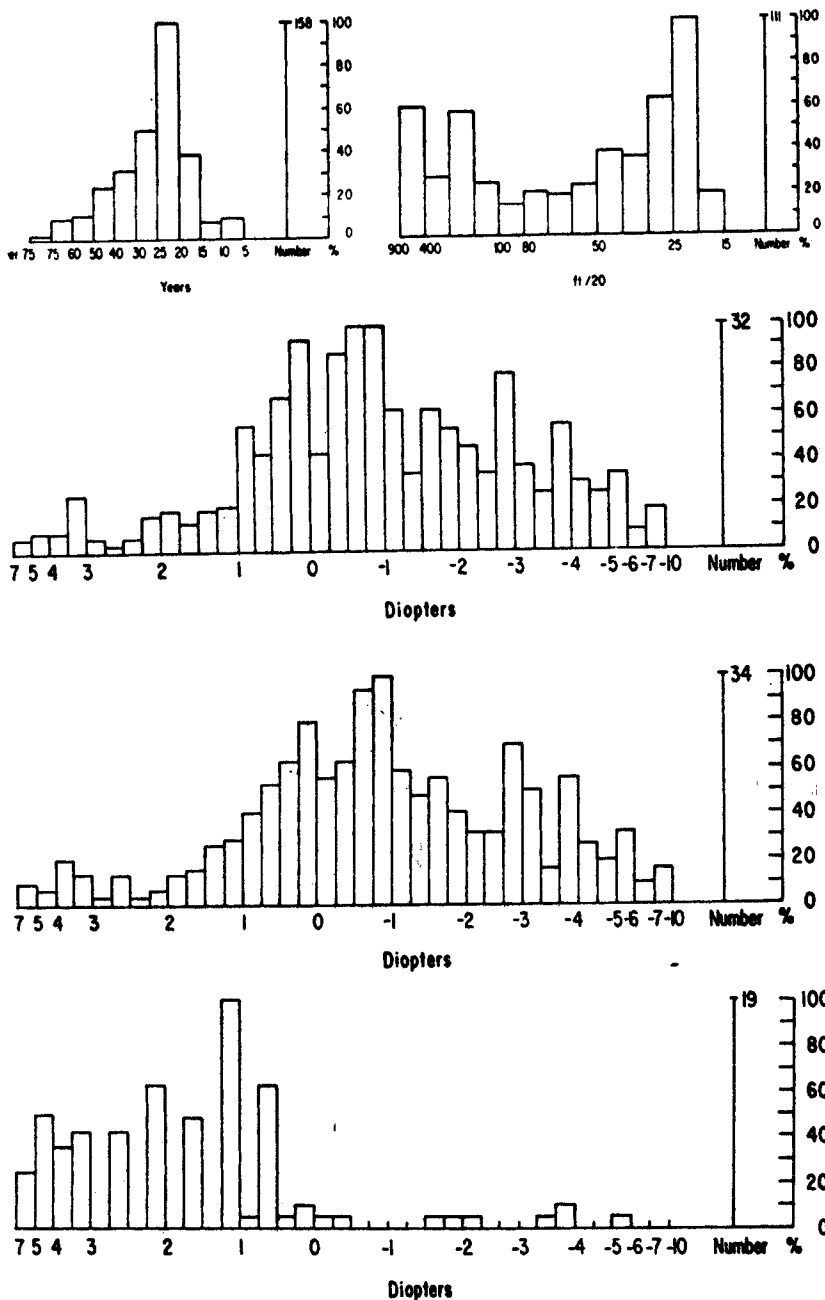


Fig. 4. Distribution of Selected Variates from 650 Consecutive Case Records. (See text for further details.) a. (Upper Left) Age in Years; b. (Upper Right) Visual Acuities, Left Eye; c. (Second from Top) Spherical Refractive Errors Determined by Subjective Refraction, Left Eye; d. (Next to Bottom) Spherical Corrections Prescribed for General Use, Left Eye; e. (Bottom) Spherical Corrections Prescribed for Reading Use, Left Eye.

Computer programs described elsewhere⁹ were used to compute entropies and information transfer rates based on formulae derived from Equations (1) – (3) above. With a total sample of only 650 message sets, the informational quantities estimated were all subject to sampling bias and random error^{6, 7, 10}.

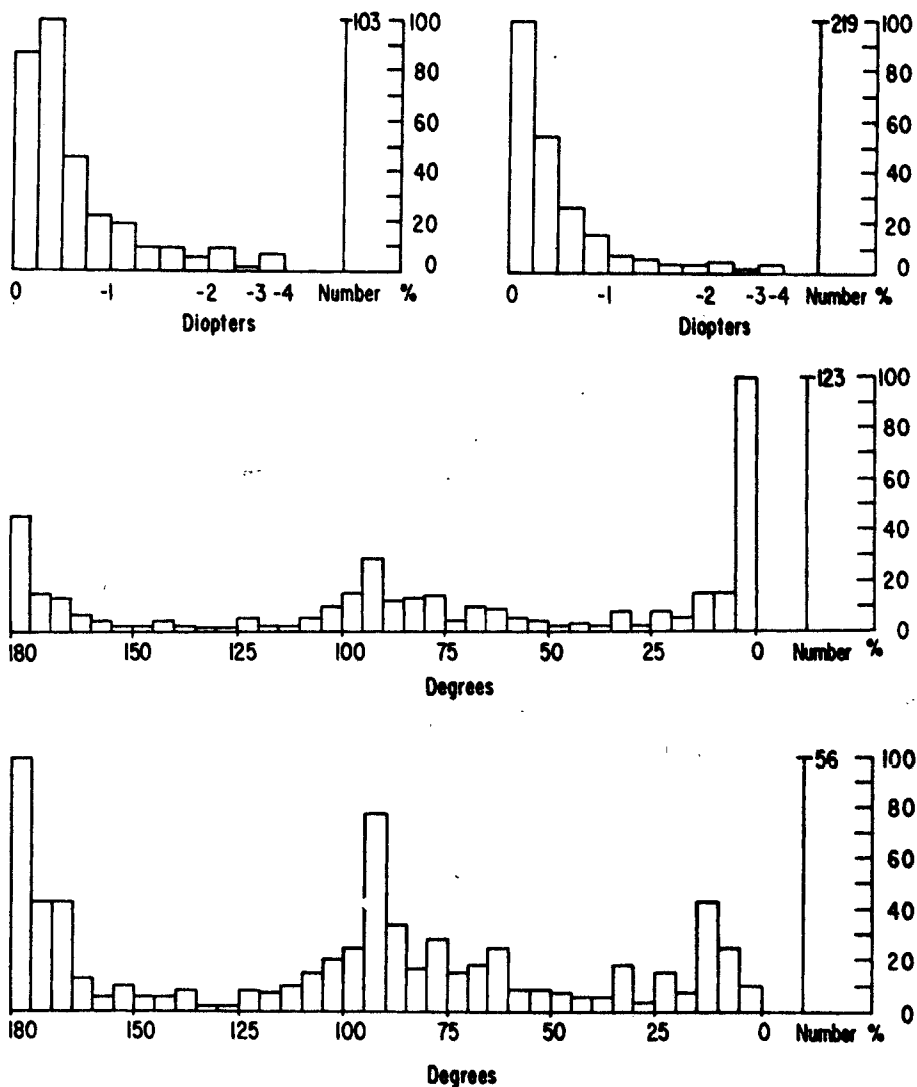


Fig. 4 (Continued) Distribution of Selected Variates from 650 Consecutive Case Records. (See text for further details.) f. (Upper Left) Cylindrical Refractive Errors Determined by Subjective Refraction, Left Eye; g. (Upper Right) Cylindrical Corrections Prescribed for General Use, Left Eye; h. (Center) Axis of Cylindrical Refractive Errors Determined by Subjective Refraction, Left Eye; i. (Lower) Axis of Cylindrical Correction Prescribed for General Use, Left Eye.

While every effort was made to control these, the reader should bear in mind that the results set out below represent sample estimates, rather than population values. Statistical significance levels were computed as indicative of the likelihood (or unlikelihood) that the quoted results could have occurred by chance, and statistically insignificant results have been suppressed.

RESULTS

Distribution Pattern for the Variables (See Table 1)

The total sample yielded histograms such as those shown in Fig. 4 for the left eye. The average age of the population was relatively low (27.85

TABLE 2
 INFORMATION SHARED BETWEEN OUTPUT VARIABLES (i.e., COMPONENTS
 OF THE TWO PRESCRIPTIONS)

		General Use Prescription						Reading Use Prescription					
		Left Eye			Right Eye			Left Eye			Right Eye		
		Spheri- cal	Cylin- dri- cal	Axis	Spheri- cal	Cylin- dri- cal	Axis	Spheri- cal	Cylin- dri- cal	Axis	Spheri- cal	Cylin- dri- cal	Axis
General Use Prescription	18 Spherical	X	—	—	—	—	—	—	—	—	—	—	—
	19 Cylindrical	0.38 (16)	X	—	—	—	—	—	—	—	—	—	—
	20 Axis		1.11 (26)	0.64 (15)	X	—	—	—	—	—	—	—	—
General Use Prescription	21 Spherical	2.32 (48)	—	—	X	—	—	—	—	—	—	—	—
	22 Cylindrical		0.49 (21)	—	0.44 (19)	X	—	—	—	—	—	—	—
	23 Axis	—	—	1.46 (34)	1.21 (28)	0.69 (16)	X	—	—	—	—	—	—
Reading Use Prescription	24 Spherical	2.61 (69)	—	—	2.32 (48)	—	—	X	—	—	—	—	—
	25 Cylindrical	—	2.34 (100)	—	—	—	—	0.73 (20)	X	—	—	—	—
	26 Axis	—	—	4.32 (100)	—	—	1.46 (34)	—	—	X	—	—	—
Reading Use Prescription	27 Spherical	2.32 (48)	—	—	2.88 (72)	—	—	2.32 (55)	—	—	X	—	—
	28 Cylindrical	—	—	—	—	2.34 (100)	—	—	0.80 (35)	—	1.00 (24)	X	—
	29 Axis	—	—	1.46 (34)	—	—	4.30 (100)	—	—	2.78 (64)	—	—	X

Figures indicate transmission values in bits. Figures in parentheses indicate the corresponding per cent exhaustiveness of transfer.

TABLE 3
INFORMATION-TRANSFER FROM INITIAL AND INTERMEDIATE VARIABLES TO OUTPUT VARIABLES (i.e., COMPONENTS OF THE TWO PRESCRIPTIONS)

		General Use Prescription						Reading Use Prescription					
		Left Eye			Right Eye			Left Eye			Right Eye		
		Spheri- cal	Cylin- drical	Axis	Spheri- cal	Cylin- drical	Axis	Spheri- cal	Cylin- drical	Axis	Spheri- cal	Cylin- drical	Axis
Non-Visual Client Data	1 Sex	0.03 (3)	0.01 (1)	0.05 (5)	*	*	*	*	*	*	*	*	*
	2 Age	0.53 (11)	0.12 (5)	0.37 (9)	0.56 (12)	0.06 (3)	0.30 (7)	1.06 (28)	0.33 (15)	1.01 (24)	1.22 (27)	0.30 (13)	1.10 (39)
Initial Visual Status	3 Visual Acuity (LE)	1.27 (27)	0.17 (7)	0.50 (13)	—	—	—	1.50 (40)	0.65 (30)	1.40 (34)	—	—	—
	4 Visual Acuity (RE)	—	—	—	1.34 (28)	0.17 (7)	0.54 (13)	—	—	—	1.46 (36)	0.61 (27)	1.81 (42)
	5 Visual Acuity (Both)	1.20 (25)	0.18 (8)	0.55 (16)	1.29 (27)	0.17 (7)	0.55 (16)	1.42 (38)	0.65 (30)	2.05 (59)	1.40 (33)	0.64 (28)	1.92 (55)
Determination of Refractive Errors Objective—Static Skiametry	6 Spherical	2.40 (50)	—	—	—	—	—	2.07 (55)	—	—	—	—	—
	7 Cylindrical	—	0.65 (25)	—	—	—	—	—	1.04 (40)	—	—	—	—
	8 Axis	—	—	1.72 (46)	—	—	—	—	—	1.97 (47)	—	—	—
	9 Spherical	—	—	—	2.50 (52)	—	—	—	—	—	2.13 (51)	—	—
	10 Cylindrical	—	—	—	—	0.76 (30)	—	—	—	—	—	1.00 (38)	—
	11 Axis	—	—	—	—	—	1.60 (33)	—	—	—	—	—	2.73 (75)
Determination of Refractive Test Subjective—Refractive	12 Spherical	3.56 (74)	—	—	—	—	—	2.03 (54)	—	—	—	—	—
	13 Cylindrical	—	1.90 (74)	—	—	—	—	—	2.10 (83)	—	—	—	—
	14 Axis	—	—	3.68 (79)	—	—	—	—	—	3.93 (91)	—	—	—
	15 Spherical	—	—	—	3.64 (75)	—	—	—	—	—	2.25 (53)	—	—
	16 Cylindrical	—	—	—	—	1.92 (75)	—	—	—	—	—	1.93 (81)	—
17 Axis	—	—	—	—	—	3.80 (71)	—	—	—	—	—	3.91 (90)	

Figures indicate transmission values in bits. Figures in parentheses indicate the corresponding per cent exhaustiveness of transfer.

*Computed and found insignificant on initial trials.

years), as may be expected in university optometric clinics. Since later results show that age transmits a significant amount of information to the final prescription, a more uniform distribution of this variable would have been preferred. The relative value of age data has probably been underestimated for this reason.

Entropy Estimates for Intermediate and Output Data

The entropy estimates for the various intermediate data and outputs, derived from the frequency distributions of Figure 4, are given in the two final columns of Table 1. It will be noted that redundancies are generally low, indicating that the available ranges of the variables are fairly fully exploited. The total intermediate entropy, or mean information available from all the selected intermediate results taken together and assumed independent, was 60.2 bits, while the total output to general use prescriptions was 22.85 bits; to reading use prescriptions, 21.15 bits. Clearly, the decision-maker of Stage 2 receives more information than the total he is required to output. Bearing in mind that general use and reading use prescriptions were themselves correlated as shown below, his informational input appeared to be considerably more than twice his output.

Common Information Between Components of the Prescriptions

Table 2 shows the estimated information transfer between the various components of the prescription. There was little common information between the three prescription components for the same eye and type of prescription; rather more was shared between the same components for opposite eyes in the same type of prescription. However, the outstanding result here was that all cylinder and axis information and about 70% of spherical information was shared as between general and reading use prescriptions. This means that an automatic system can be programmed to generate reading use prescriptions simply by repeating cylindrical and axis data, together with minor adjustment to the spherical data; the additional information required amounts to rather less than one bit per reading use prescription per eye. The corresponding information-transfer model is shown in Fig. 5.

Information Transfer from the Intermediate Data to Prescription

Table 3 summarizes the information transferred from the various non-

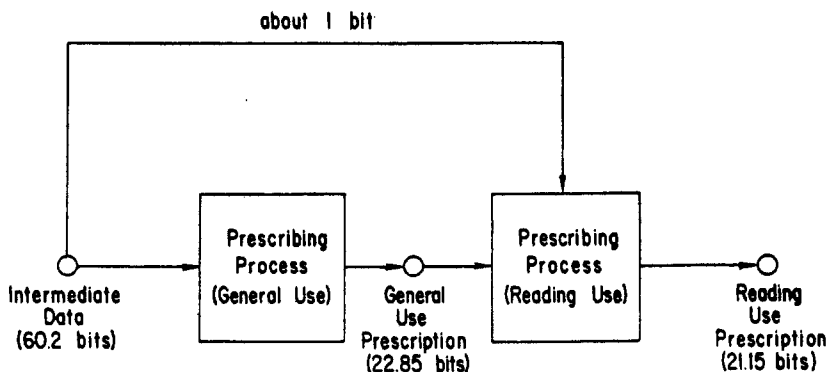


Fig. 5. Information Transfers Estimated from a Sample of 650 Consecutive Cases. Tandem Placement of General Use and Reading Use Prescription.

visual and initial visual status variables to the two final prescriptions separated by component. Statistically insignificant entries have been suppressed, and the exhaustiveness^c, that is the transferred information as a per cent of output, is shown in parentheses. It will be noted that while small amounts of information are transferred from initial visual status and from visual acuity to the general use prescription, and moderate amounts from static skiametry to both general use and reading use prescriptions, much the highest transfers are from specific components of subjective refraction to like components of reading and general use prescriptions. The average exhaustiveness here was 74.7%, with missing information (the amount by which output information exceeds transferred information) averaging about one bit per dimension (see Table 4). Static skiametry shows up much less well with missing information averaging nearly 2.4 bits per dimension. We conclude that if an automatic procedure reproducing the intermediate data obtained by subjective refraction were imple-

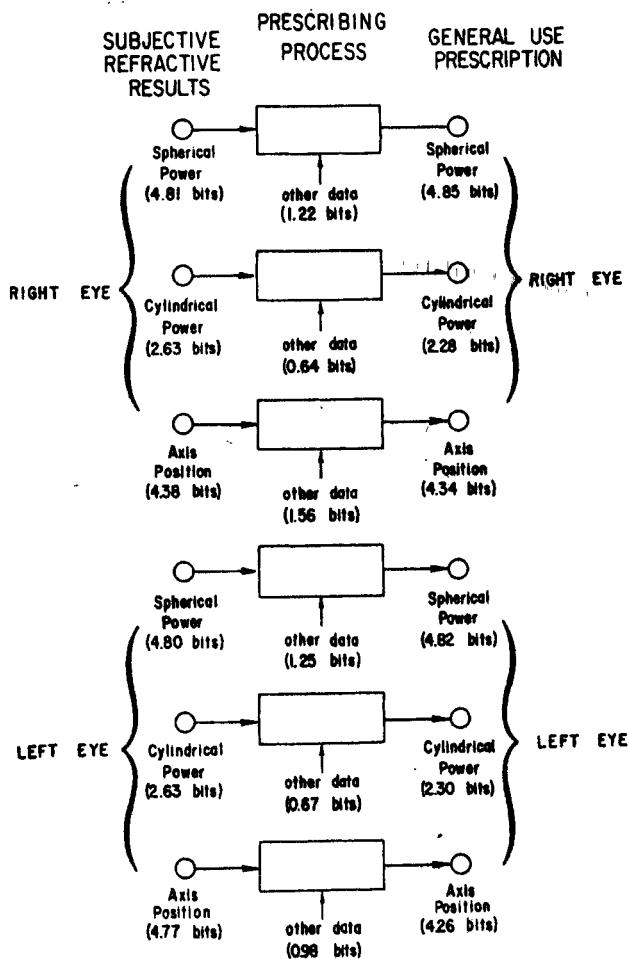


Fig. 6. Information Transfers Estimated from a Sample of 650 Consecutive Cases. Parallel-Channel Model Generating Components of the General Use Prescription.

^cCorresponding roughly to a coefficient of association.

mented, the components of its output could be converted directly into a prescription with little loss of precision relative to an optometrist's results.

TABLE 4
 INFORMATION IN OUTPUT VARIABLES UNEXPLAINED BY TRANSFER FROM
 SINGLE INTERMEDIATE VARIABLES

Dimension of Prescription	Given Results of Subjective Refraction Extra Information Needed to Prescribe Is	Given Results of Static Skiametry Extra Information Needed to Prescribe Is
Spherical Power, Right Eye	1.22 bits	2.31 bits
Spherical Power, Left Eye.....	1.25 bits	2.40 bits
Cylindrical Power, Right Eye.....	0.64 bits	1.78 bits
Cylindrical Power, Left Eye	0.67 bits	1.96 bits
Axis Position, Right Eye	1.56 bits	3.74 bits
Axis Position, Left Eye	0.98 bits	2.02 bits
Total Amount of Missing Information.....	6.32 bits	14.21 bits
Mean per Dimension.....	1.05 bits	2.37 bits

From a modelling standpoint, this result tends to validate the independent-channel model of Fig. 3b, and further indicates that only about twelve of the possible 204 intermediate-data-to-output-component channels transfer significant amounts of information. Since six of these are redundant with regard to the required output data, our final model-form is that of Fig. 6.

Joint Transfer of Information from Pairs of Intermediate Data to Prescription

While the results quoted above show that subjective refraction can alone provide most of the information needed for a general use, and hence a reading use prescription, the missing 1.05 bits per component per eye (Table 4) should ideally be identified. It is entirely possible that this simply represents "noise," such as might be introduced by errors in recording of intermediate data or prescriptions, or from aberrations of the optometrist's judgment. If no coherent effect were found to explain the difference between prescription and the results of subjective refraction, we would be forced to accept this alternative.

A search was therefore conducted for the missing information, using the technique of multi-dimensional information transfer. By this technique, we can estimate the information transferred jointly from two input messages to a single output, with the corresponding exhaustiveness as defined above. Many such estimates were computed, and only selected significant results are given in Table 5. From these it appears that inclusion either of age or of visual acuity data could significantly improve the quality of an automatically-generated prescription, leaving an average of only 0.75 bits of missing information per component per eye. The interesting result also emerges that age and initial visual acuity can alone (i.e., without any form of refraction procedure) provide nearly half the information needed to produce a prescription. Computational complexity prevented us from examining three and more input to single output information transfers, which should have ideally been considered. Hence we must consider the missing 0.75 bits as noise.

CONCLUSIONS

The informational analysis has established the following conclusions rela-

TABLE 5
INFORMATION IN OUTPUT VARIABLES TRANSFERRED FROM PAIRS OF INITIAL AND INTERMEDIATE VARIABLES

From Variable	Entropy Bits	And Variable	Entropy Bits	To Variable	Entropy Bits	Transferred Information (Bits)	Exhaustiveness	Missing Information (Bits)	Sample Size
Visual Acuity (Left Eye)	3.50	Age	2.70	General Use Prescription Spherical Error (Left Eye)	4.83	2.47	51%	(2.36)	437
Subjective Refraction Spherical Error (Left Eye)	4.81	Age	2.69	General Use Prescription Spherical Error (Left Eye)	4.82	4.11	85%	0.71	435
Subjective Refraction Spherical Error (Left Eye)	4.81	Visual Acuity (Left Eye)	3.50	General Use Prescription Spherical Error (Left Eye)	4.82	4.03	84%	0.79	432
Visual Acuity (Left Eye)	3.49	Age	2.71	General Use Prescription Cylinder Error (Left Eye)	2.33	0.75	32%	(2.37)	431
Subjective Refraction Cylindrical Error (Left Eye)	2.67	Age	2.67	General Use Prescription Cylinder Error (Left Eye)	2.57	2.07	81%	0.50	314
Subjective Refraction Cylindrical Error (Left Eye)	2.62	Visual Acuity (Left Eye)	3.49	General Use Prescription Cylinder Error (Left Eye)	2.58	2.10	81%	0.48	311
Visual Acuity (Left Eye)	3.49	Age	2.71	General Use Prescription Axis Position (Left Eye)	4.24	1.85	43%	(2.39)	430
Subjective Refraction Axis Position (Left Eye)	4.65	Age	2.67	General Use Prescription Axis Position (Left Eye)	4.63	4.06	88%	0.57	314
Subjective Refraction Axis Position (Left Eye)	4.65	Visual Acuity (Left Eye)	3.49	General Use Prescription Axis Position (Left Eye)	4.62	4.22	91%	0.40	311

≡ 1.78 bits (14.8%)
 ≡ 1.67 bits (13.9%)
 ≡ 7.12 bits (62.5%)

Total missing information for left eye after taking age into account
 Total missing information for left eye after taking visual acuity into account
 Total missing information for left eye using age and visual acuity only

tive to the proposed automatic system:

- 1) The system must provide respectively 4.8, 2.3, and 4.3 bits of spherical power, cylindrical power, and axis position for general use prescription information per patient per eye.
- 2) Nearly all of this can be obtained by replicating the manual sub-

jective refraction process.

3) The reading use prescription requires relatively little additional spherical power information.

4) Visual acuity and age can each contribute small but definite amounts of missing information to the final prescription.

We hope that an opportunity will arise to validate this analysis by direct comparison of automatically-produced prescriptions with parallel manually-produced ones.

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